

THE GIFFORD-MCMAHON CYCLE

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ABSTRACT

The method of producing refrigeration at low temperatures by what has become known as the Gifford-McMahon cycle is described in considerable detail. Questions of whether or not it is a cycle and how it would appear on a conventional temperature-entropy diagram are discussed. The analysis shows that the cycle efficiency differs little from that of the Brayton cycle at low pressure ratios. A new device designed to take full advantage of the practical simplicities afforded by the Gifford-McMahon cycle is also described.

THE GIFFORD-MCMAHON CYCLE

In 1959 a new refrigeration method was introduced which has seen considerable application in the making of small cryogenic refrigerators; see references (1) through (7). The thermodynamic cycle employed in this refrigeration method has become known as the Gifford-McMahon Cycle. Since only rather incomplete thermodynamic analyses have been given in the past and many individuals have incorrect ideas about its possible efficiency, an attempt will be made here to give a more extensive description and analysis.

The question might arise as to whether it is a cycle or not. A cycle is defined as follows, (8): "When a system in a given state goes through a number of different processes and finally returns to its initial state, the system has undergone a cycle." In the Gifford-McMahon Cycle all the gas does not proceed through the same series of processes. Here each little batch of gas goes through a different series of processes but does return to the initial state. The net result is the sum of the actions of all the different batches of gas which act differently but return to the same initial state. Since the definition of a cycle does not preclude this possibility, the Gifford-McMahon Cycle may legitimately be called a cycle.

The fact that many little segments of the operating gas act differently causes a problem when an attempt is made to describe it thermodynamically, such as by plotting it on a Temperature-Entropy diagram. The processes followed by systems operating on other cycles, such as turbines operating on the Brayton cycle, can be described by a single line on a T-S diagram. A single area is enclosed and the areas under lines have special significance. The Gifford-McMahon Cycle, however, involves a continuum of lines, thus giving areas which overlap. To get a clear idea of the cycle through reference to a T-S diagram it is necessary to plot several different T-S diagrams for a selection of different batches of gas which show the different processes which are occurring.

The reasons for the Gifford-McMahon Cycle being useful are of a practical

nature, such as simplicity, reliability and ease of construction. There is, however, a small sacrifice in efficiency compared with the Carnot cycle. It is interesting that in making the constructional details simple, as compared to the Sterling cycle devices, one makes the thermodynamic analysis aspects more complex.

In order to make an analysis of the Gifford-McMahon Cycle it is necessary to have a model to refer to as we show the processes followed by different batches of gas. This is shown by the schematic diagram, Figure 1, for a new method, Cryomatic Gas Balancing⁽⁵⁾, which applies the Gifford-McMahon Cycle with a minimum of device complexity. The method achieves refrigeration in a system composed of a volume divided into several chambers by a free-floating displacer. Motion of the displacer is caused by the unbalance of pressures in these chambers resulting from the delivery and exhaustion of gas through the simple rotary cored valve.

As is shown, the closed volume created by the two coaxial cylinders of different size is divided into three variable chambers by the free-floating displacer. The size of each chamber depends on the position of the displacer. Chamber (2) at the intersection of the two cylinders and Chamber (3) at the end of the large cylinder are interconnected by a small, highly efficient thermal regenerator of small pressure drop so that both of these chambers are always maintained at approximately equal pressure. A suitable seal separates Chamber (1) from Chamber (2) so that the pressure in Chamber (1) can be different from that in Chambers (2) and (3). A rotary cored valve is used to raise the pressure by delivering high pressure gas from a compressor, or to lower the pressure by exhausting to the low pressure line from either Chamber (1) or Chambers (2) and (3). The timing between pressurizing and depressurizing the chambers may be set by the location of the parts connecting the chambers with the valve.

At the beginning of the series of operations, the displacer is in its topmost position so that the volumes of Chambers (1) and (2) are zero and Chamber (3) is at a maximum volume, with the rotating valve reference X at 11:00 o'clock. Pressure

is low in all three chambers since the low pressure slot is opposite both connecting tubes.

When the valve rotates counterclockwise so that reference X passes 9:00 o'clock, it allows high pressure gas to be delivered to Chamber (1) only, forcing the displacer to move to the bottom position. Gas is transported from Chamber (3) to Chamber (2). If Chamber (3) is colder, some gas will be exhausted through the valve as a result of expansion back to the inlet temperature.

When the rotating valve (reference X) passes 6:00 o'clock, high pressure gas is let into Chambers (2) and (3). This builds the pressure there to the high pressure while the high pressure is still maintained in Chamber (1). The displacer does not move.

When the rotating valve (reference X) passes 3:00 o'clock the pressure drops in Chamber (1) because of the movement of the low pressure cored slot to a position in register with the Chamber (1) connecting tube. This causes an unbalance of pressure on the displacer so that it moves to its topmost position transferring the gas in Chamber (2) at constant high pressure to Chamber (3). If Chamber (3) is colder, as it will be in operation, additional high pressure gas will be supplied through the valve during this transfer.

When the rotating valve (reference X) passes 12:00 o'clock, it allows the pressure in Chambers (2) and (3) to drop to a low value because the low pressure cored slot in the valve is now opposite the Chamber (2) and (3) connecting tubes. Chamber (1) retains its low pressure as the low pressure slot is still opposite its connecting tube, so this process takes place without the displacer moving. This completes the series of operations which is repeated with each revolution of the valve.

The Cryomatic Gas Balancing refrigerator is quite an easy thing to build, as it involves only two slow-moving parts. Operating speed is only about 50 to 150 cycles per minute. It can make an excellent refrigerator. Temperatures as low as

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23°K have been reached in Chamber (3) with a 1¼" diameter displacer operating at 120 RPM.

It is relatively simple to make a multi-stage unit (Figure 2) where refrigeration is developed at two different low temperatures. Only one additional moving part, the displacer, and an additional thermal regenerator are required. Several such machines have been built which achieve temperatures as low as 12°K. An actual two-stage experimental refrigerator of this type is shown in Figure 3. It has 5 watts refrigeration capacity at 15°K and 18 at 50°K and has served as the base for a 4.2°K refrigerator.

The general nature of the cycle may be shown on a T-S diagram by following what happens to a small part of the gas involved. In Figure 4 is shown a T-S diagram for the first gas to enter the refrigerator which contributes to the refrigeration achieved. It enters the valve at room temperature, T_R , point 1, and expands through the valve isenthalpically to very nearly the low pressure, P_L , point 2. It then is transferred into Chamber (2) with only a slight pressure increase. There it is compressed back to the high pressure, P_H , point 3, by the addition of more gas.

In the act of being transferred out of Chamber (2) when the displacer moves up it will be mixed with additional gas from the supply and comes to a lower temperature, T_c , point 4. It is then cooled in the regenerator to the cryogenic temperature, T_R , point 5.

After Chamber (3) has reached its maximum volume the valve next allows this cold high pressure gas to be exhausted. The expansion before it leaves Chamber (3) allows it to drop slightly in temperature to 6, giving the refrigeration effect, before being heated again in the regenerator to 7 and passing through the valve expanding isenthalpically to the low pressure, point 8, at a temperature close to that of the mixing temperature. Compression and cooling to pass from 8 to 1 is not shown on the diagram as it would obscure the clarity of the other lines. The complete cycle, of course, involves this also.

It is to be remembered that this only represents the action for the first small fraction of gas to enter the refrigerator. Every other small fraction of gas will have different diagrams. For example, gas that enters the refrigerator after the pressure is about half up will follow the diagram of Figure 5 where numbers have the same significance as in Figure 4. All of the gas that is added at the high pressure during the period when the displacer is moving up does not proceed through the processes 1 to 4 but mixes directly with the initial pressurizing gas, proceeding from point 1 directly to point 4. A complete set of lines for all segments of the gas involved would give many overlapping areas. It thus is a very difficult cycle to put on a T-S diagram.

The reason one prepares any diagram showing any functional relationship, series of operations, or a cycle, is for the purpose of making it more understandable by a simple direct picture. A T-S diagram is selected for many cycles because it makes it possible to see clearly the overall operation quickly and simply including pictorially the heat and work quantities involved. A T-S diagram does not accomplish this for the Gifford-McMahon cycle. It makes the cycle more obscure by presenting a rather difficult intellectual problem. Therefore, there is really no good reason for attempting to put it on a T-S diagram. It is rather like solving a heat transfer problem, involving a rectangular block, in spherical coordinates.

The best method for computing the performance of the Gifford-McMahon Cycle has been shown in previous papers ⁽¹⁾⁽²⁾⁽³⁾ by consideration of what is going on in the chambers in an overall way, rather than integrating the effects of all the different batches of gas with varying actions.

It has been shown that the total refrigeration, Q , is

$$Q = \int V dP = \int P dV$$

for the expansion space or

$$Q = V (P_h - P_l) \quad (1)$$

where V is the volume of Chamber (3), P_h the high pressure, and P_l the low pressure.

In the analysis of a refrigeration cycle, the desired result is an ability to compute the ideal performance of a proposed system and then also show how each factor which detracts from this ideal performance affects the total performance. For a refrigerator the thing you want to know is how much work, W , at room temperature, T_R , is required to remove an amount of heat, Q , at a low temperature, T_r .

The work required to compress a given mass of gas, m , in an adiabatic process is

$$W = m C_p T_R \left[\left(\frac{P_h}{P_l} \right)^{(\gamma-1)/\gamma} - 1 \right] \quad (2)$$

where C_p is the specific heat of the gas at constant pressure, γ , the ratio of specific heats, and T_R , the temperature of the gas entering the compressor.

The mass of gas used in the Gifford-McMahon Cycle would be the difference between the mass of gas in Chamber (3), m_3 , at the low temperature T_r and high pressure P_h , and the mass of gas in Chamber (2) at temperature, T_c , and at the low pressure P_l . In an actual system the volume of Chamber (1) is small relative to Chamber (2) and little accuracy is lost if it is assumed to be so small that Chamber (2) is equal in volume to Chamber (3).

$$m = m_3 - m_2 \quad (3)$$

Assuming the perfect gas relation is valid,

$$m = \frac{P_h V_3}{R T_r} - \frac{P_l V_2}{R T_c} \quad (4)$$

where R is the gas constant.

The work per cycle is thus

$$W = C_p T_R \left(\frac{P_h V_3}{R T_r} - \frac{P_l V_2}{R T_c} \right) \left[\left(\frac{P_h}{P_l} \right)^{(\gamma-1)/\gamma} - 1 \right] \quad (5)$$

Dividing Equation (5) by (1)

$$\frac{W}{Q} = \frac{C_p T_R \left[(P_h V_3 / R T_R - P_l V_2 / R T_c) \right] \left[(P_h / P_l)^{(\gamma-1)/\gamma} - 1 \right]}{V_3 (P_h - P_l)} \quad (6)$$

The assumption that Chamber (2) is equal to Chamber (3) and that $T_R/T_c \approx 1$ allows the relation to simplify to

$$\frac{W}{Q} = \frac{C_p \left[(P_h / P_l) (T_R / T_R) - 1 \right] \left[(P_h / P_l)^{(\gamma-1)/\gamma} - 1 \right]}{R (P_h / P_l - 1)} \quad (7)$$

The work per unit heat removed at a low temperature, W/Q , increases as the temperature ratio increases, just as it does for Brayton cycle refrigerators. For T_R of 300°K and T_c of 80°K , Figure 6 shows a plot of W/Q as a function of pressure ratio from Equation (7) assuming helium is the refrigerant. Also shown on the same plot is the relationship for the Carnot cycle and the Brayton cycle, with and without work recovery. The relation for the Carnot cycle is well known as

$$W/Q = (T_R - T_c) / T_c \quad (8)$$

and the relation for the Brayton cycle is

$$W/Q = (T_R / T_c) (P_h / P_l)^{(\gamma-1)/\gamma} - 1 \quad (9)$$

The Brayton cycle is really the ideal cycle for the refrigerators that are said to be Sterling cycle refrigerators. The Sterling cycle calls for isothermal compression and expansion, and the Brayton cycle isentropic compression and expansion. The .01 second allowed for compression and expansion in the Sterling cycle devices does not give time for the transfer of an appreciable amount of heat during these processes. It is not sufficient to cool after compression. To achieve Sterling cycle efficiency the cooling must occur during the compression.

The Brayton cycle is also the ideal cycle of small expansion engine or turbine refrigerators, if the work of the expansion device is not recovered. If not, the ratio of W/Q is given by Equation (9) with the one deleted.

The Gifford-McMahon Cycle as seen from the curves requires more work for a given amount of refrigeration than the Brayton cycle. However, the difference is not great for small compression ratios which is the way any refrigerator designed to use these cycles should be operated. It is interesting to note that a Gifford-McMahon cycle at a pressure ratio of 2 is about equivalent to a Brayton cycle at a pressure ratio of 3.5. Gifford-McMahon Cycle devices work very well with pressure ratios of 1.5 to 2.5. There is no need to go to higher pressure ratios and thus poorer efficiencies. The Gifford-McMahon Cycle does sacrifice some potential efficiency for its many practical advantages; this sacrifice, however, is not very great.

The practical advantages, as have been described in previous papers, ⁽¹⁾⁽²⁾⁽³⁾ are numerous. They all should lead to high reliability because the basic equipment components are simple, slow moving, and not highly stressed components. This is especially true for the new method, Cryomatic Gas Balancing ⁽⁵⁾ which applies the Gifford-McMahon Cycle with only two slow moving parts. The gas seal on the displacer moves at the rate of 2 to 5 inches per second rather than 30 to 50 inches per second as in some small cryogenic refrigerators. This should make continuous operating times of 5,000 to 10,000 hours a probability.

An added advantage of the new method is the relative simplicity with which a different model of new capacity may be developed. There are only a few basic parts which are all relatively easy to construct so that prototype model development costs may be greatly reduced.

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NOMENCLATURE

P	=	Pressure
Q	=	Heat quantity
V	=	Volume
T	=	Temperature
S	=	Entropy
X	=	Position
W	=	Work
m	=	Mass
C_p	=	Specific heat at constant pressure
γ	=	Ratio of specific heats
R	=	Gas constant

Subscripts

h	high
<i>l</i>	low
R	room
r	refrigeration condition
C	Chamber 2 condition
1	Chamber 1 (see figure 1)
2	Chamber 2 "
3	Chamber 3 "

Superscripts

'	Room temperature condition
"	Low temperature condition

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- Figure 4 Temperature-Entropy Diagram for first batch of gas to enter refrigerator
- Figure 5 Temperature-Entropy Diagram for batch of gas entering the refrigerator when the pressure has been built up half way
- Figure 6 Refrigeration Cycle Performance Curves

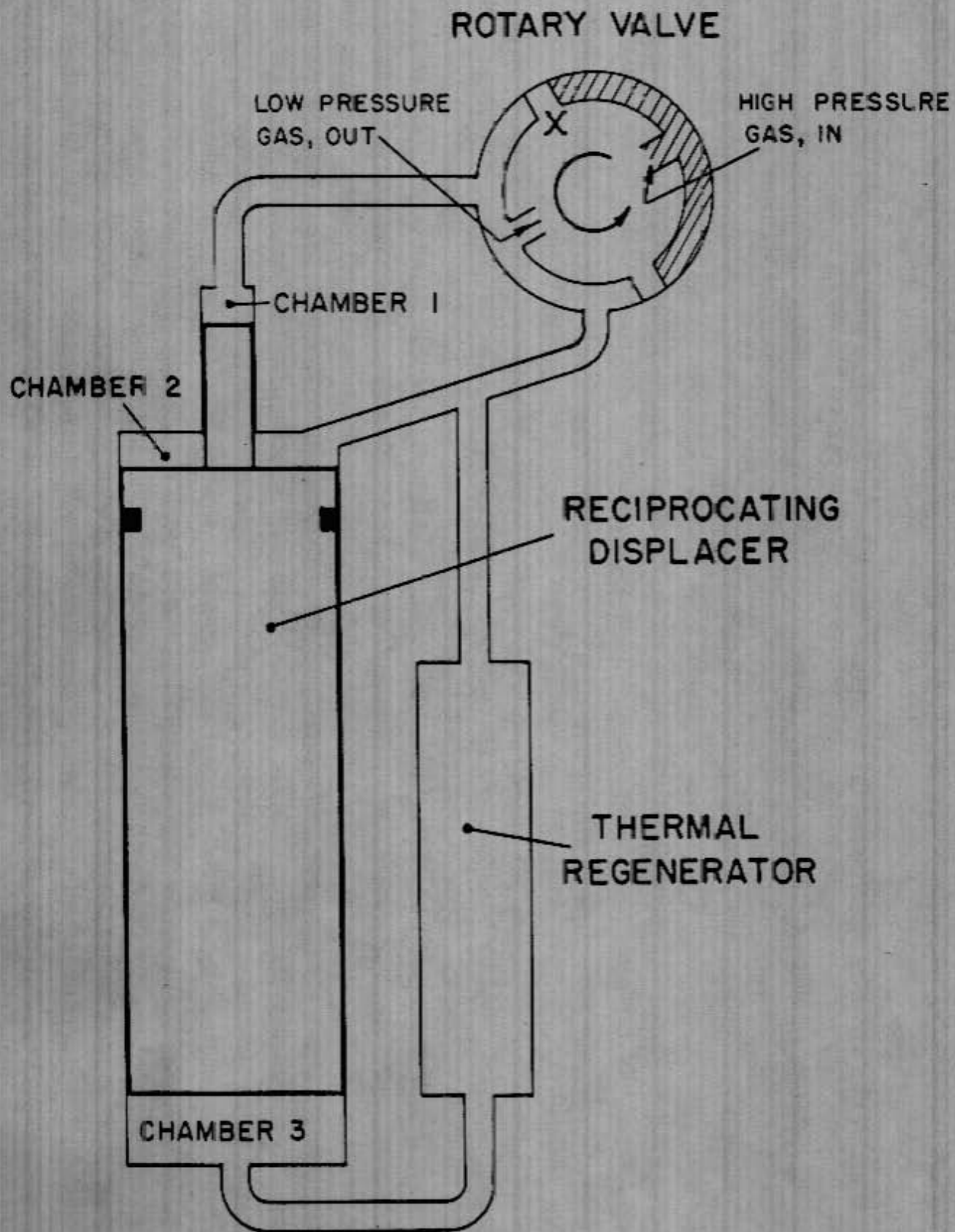


FIGURE 1. Single Stage Cryomatic Gas Balancing Refrigerator, Schematic Diagram

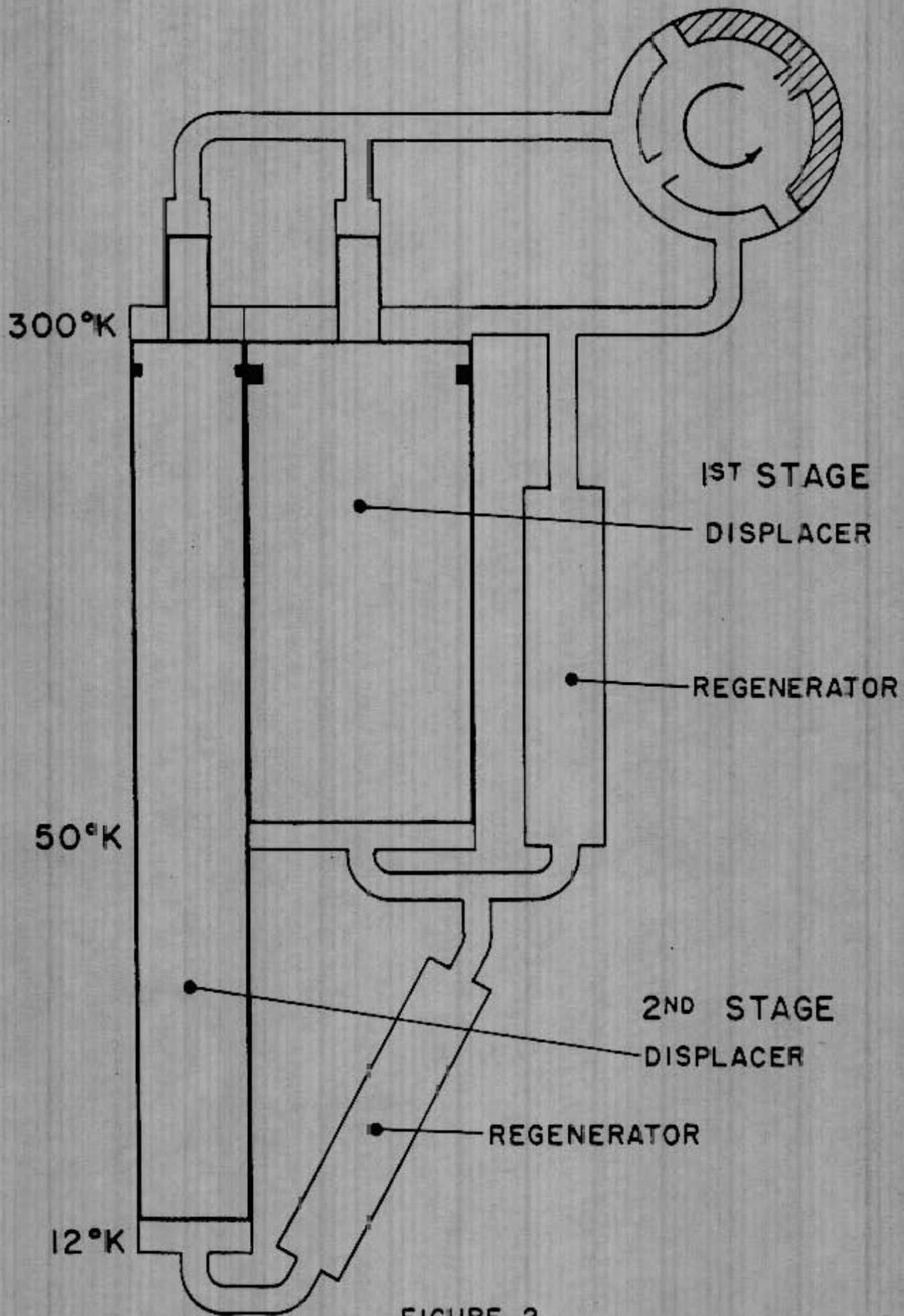


FIGURE 2.

Two Stage Cryogenic Gas Balancing Refrigerator,
Schematic Diagram

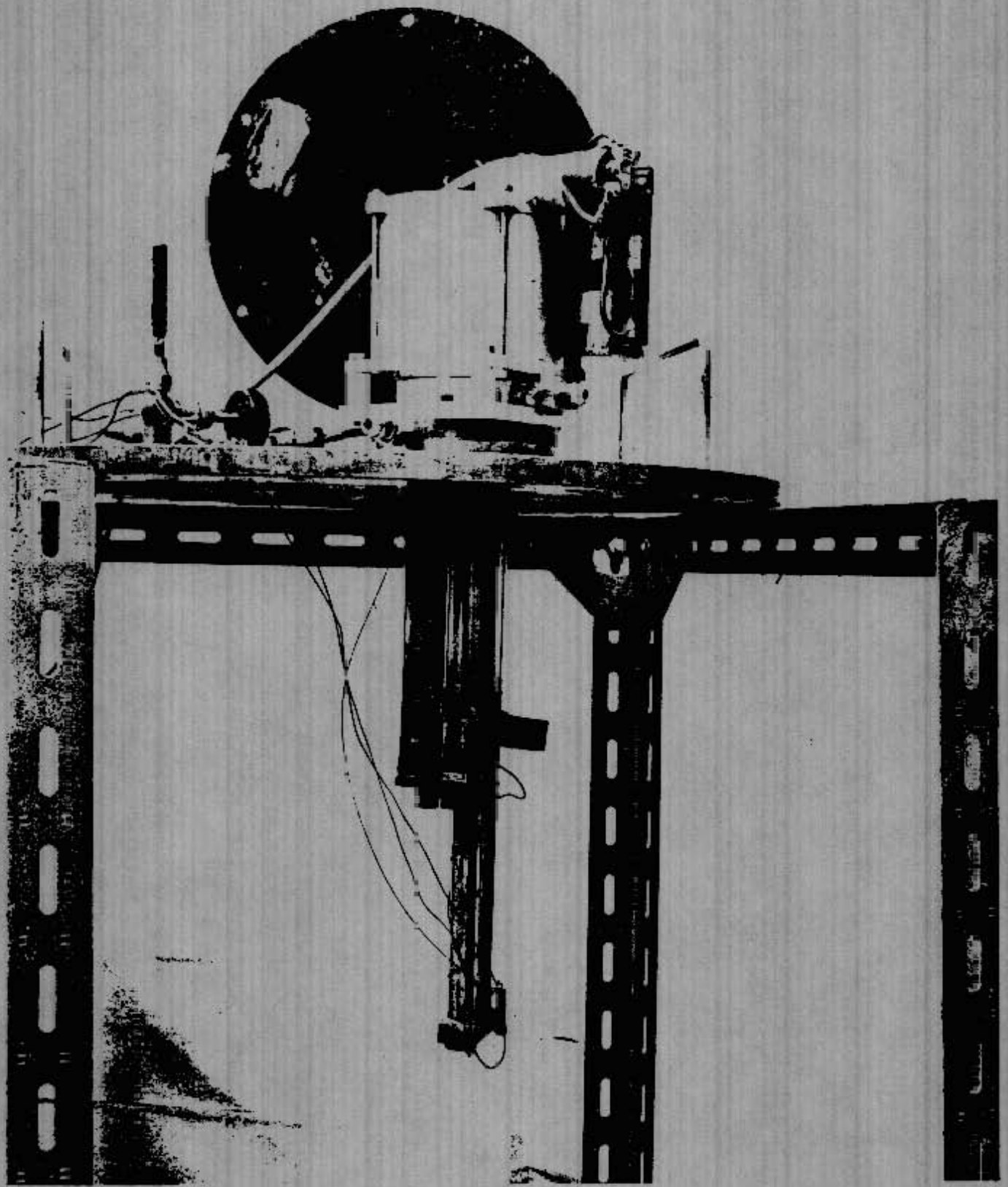


Figure 3 Two Stage Gas Balancing Refrigerator Model

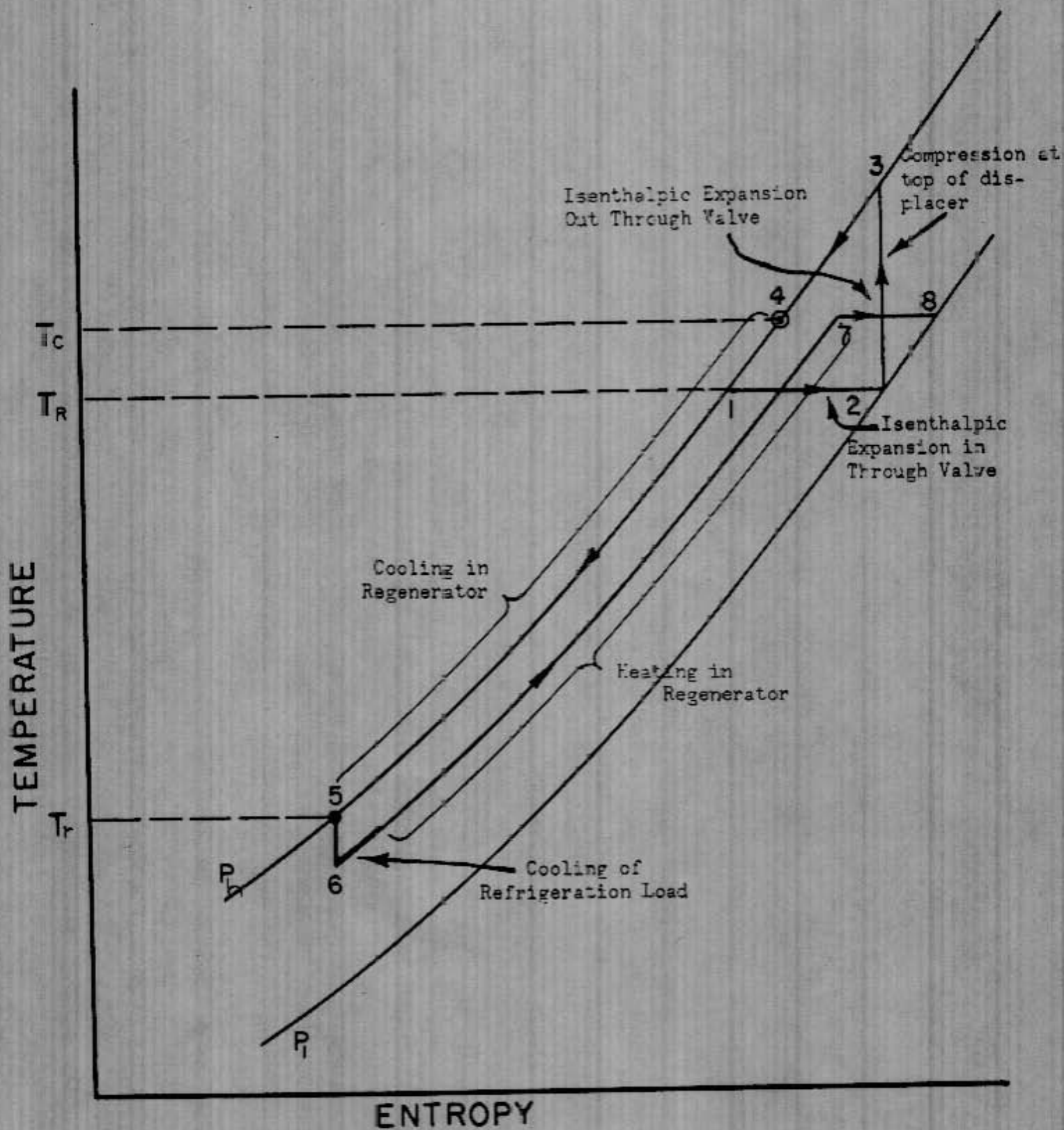


FIGURE 4. Temperature-Entropy Diagram for first batch of gas to enter refrigerator

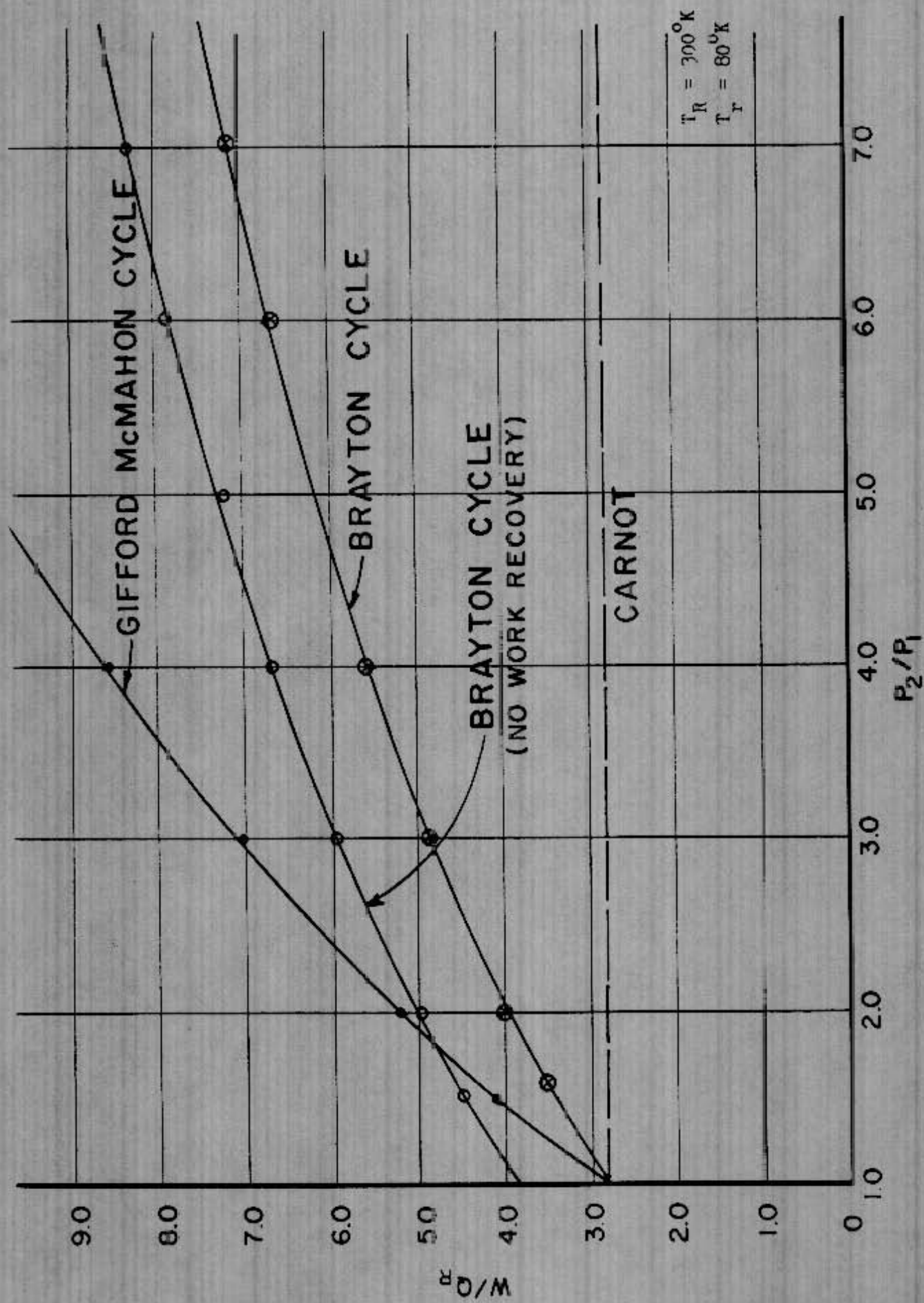


FIGURE 6. Refrigeration Cycle Performance Curves